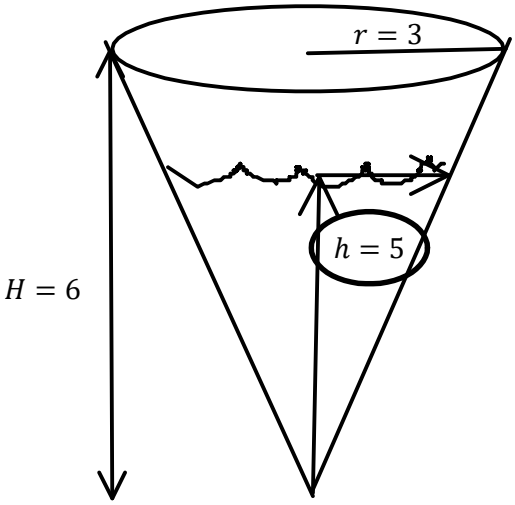
C12 - 4.3 - Cone V/Similar Triangles Related Rates Notes

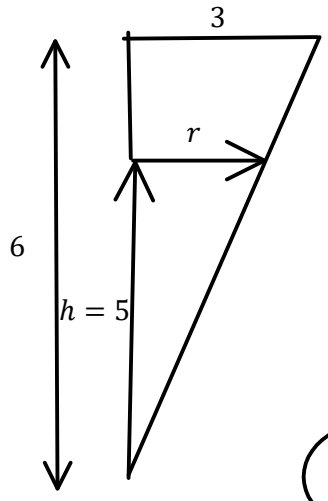
Find the rate of change.

A cone with a radius of 3 cm and height of 6 cm is filling with water where the height of the water level is increasing at a rate of 0.2 cm/s. What is the rate the volume is increasing when the height of the water level is 5 cm.



$$\frac{dh}{dt} = 0.2$$

$$\frac{dV}{dt}|_{h=5} = ?$$



$$\frac{H}{R} = \frac{h}{r}$$
$$\frac{6}{3} = \frac{h}{r}$$
$$2 = \frac{h}{r}$$
$$r = \frac{h}{2}$$

$$V = \frac{1}{3}\pi r^2 h$$
$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$
$$V = \frac{1}{12}\pi h^3$$
$$\frac{dV}{dt} = 3 \times \frac{1}{12}\pi h^2 \frac{dh}{dt}$$
$$\frac{dV}{dt} = \frac{1}{4}\pi(5)^2(0.2)$$
$$\frac{dV}{dt} = \frac{5\pi}{4}\frac{cm}{s}$$

*We can't take this product so we must use similar triangles/other info

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + \frac{dh}{dt}r^2\right)$$